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Powered Earth–Mars Cycler with Three-Synodic-Period Repeat Time

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We construct a cycler (with acceptable transfer times and moderate encounter velocities) by patching a series of three-synodic-period semicycler trajectories together. The cycler employs high-efficiency low-thrust propulsion for trajectory maintenance and correction (thus making it a "powered" cycler). Even though the propellant usage is not insignificant, we believe that this cycler still compares favorably with ballistic cyclers (that is, cyclers that do not require deterministic maneuvers), which require four (or more) vehicles, especially when considering the long-term cost to supply and maintain each vehicle. An attractive feature of this cycler is that both short inbound and outbound legs occur within each semicycler segment, thus reducing the number of required vehicles (which provide transfer opportunities every synodic period) to only three.

Introduction

LET us suppose that there is a settlement on Mars and groups of settlers from Earth are now calling Mars their home. Most likely Mars will not provide everything they need to survive, and so some necessities must come from Earth. In addition, personnel and scientific samples on Mars will be transported back to Earth. Thus we can assume that there will be frequent, two-way traffic between the two planets.

The simplest way to accomplish this flow of traffic is to launch goods and crews directly from one planet to another. Using this method, a spacecraft will leave the home planet (Earth), travel to and eventually land on the destination planet (Mars). When a return trip is desired, another spacecraft leaves Mars and comes back to Earth in the same manner. Over time, the cost to maintain, repair, and relaunch these interplanetary vehicles will be substantial.

Alternative methods exist. Instead of landing on the destination planet each time, we can place a spacecraft on a trajectory that cycles back and forth between Earth and Mars. A trajectory that circulates between two or more planets indefinitely (without deterministic maneuvers) is called a ballistic cycler. Once placed on a ballistic cycler, the spacecraft might need to perform periodic trajectory-correction maneuvers to maintain the orbit, but because the vehicle never lands on Earth or Mars we only need to launch the spacecraft once. We refer to cases where deterministic thrusting is required to maintain the cycler trajectory as "powered" cyclers.[1] We will consider the use of solar electric propulsion in our design of powered cyclers. In such a propulsion system, the specific impulse can be as high as 3300 s. Several cycler trajectories have already been discovered and analyzed; many of them employ gravity assists to reshape and reorient the orbits at each planetary encounter.[1–12]

At each planetary flyby, the transfer between the surface and the cycler spacecraft is accomplished through hyperbolic rendezvous with smaller vehicles called "taxis." The design of hyperbolic rendezvous for Earth–Mars cycler missions has been studied in detail by Nock.[10,11] Our work in this paper concentrates on the design of the cycler trajectory alone.

The most well-known cycler trajectory is the Aldrin cycler, proposed by Aldrin in 1985 (Ref. 6). The Aldrin cycler[7] can provide fast transfers to Mars (in which case it is called the outbound cycler) or fast transfers back to Earth (in which case it is called the inbound cycler). Two cycler vehicles, one on each cycler trajectory, would allow a visit to each planet every Earth–Mars synodic period (about 2.14 years). The transfer time is typically less than six months. The main disadvantage of the Aldrin cycler is the moderate to high flyby V_∞ at Mars (ranging from about 7 km/s to nearly 12 km/s). High V_∞ can make taxi rendezvous with the cycler vehicle very costly.[10]

Another type of circulating trajectory is the semicycler. These trajectories are similar to the cyclers, with one main difference: the semicyclers remain in an orbit about Mars for a period of time before returning to the Earth, whereas the cyclers perform only flybys at each planet.[9,13] Because the semicycler is placed in an orbit about Mars, the taxi rendezvous is less costly because the V_∞ are effectively zero. The main disadvantage of the semicyclers is the propellant cost for planetary captures and escapes.

Designing cyclers is generally more difficult than designing semicyclers (which are free-return trajectories connected by parking orbits about Mars) because of the more stringent timing constraints. If we replace the orbit insertion about Mars by a phasing orbit about the sun, we can "patch" together series of semicyclers to form a continuous orbit: a cycler. The resulting trajectory combines the advantages of the cyclers and the semicyclers, while lessening the undesired features of both.

In this paper, we explore an alternative to the existing cyclers in the literature. In particular, the high V_∞ of the Aldrin cycler at Earth and Mars will require greater propellant mass (or greater heat-shield mass for aerocapture) for the taxis (than for cyclers with lower V_∞). High V_∞ is considered to be a disadvantage, although this disadvantage is significantly dependent upon the mass ratio of the taxi vehicle to the cycler vehicle. If the mass ratio is small, then the taxi cost is of less concern. Also, the number of cycler vehicles can represent a substantial cost. The design study by Nock[11] based on the Aldrin cycler indicates that the overall cost is insensitive to the V_∞ of the cycler vehicles. Other designs, with different taxi/cycler mass ratio

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and other design assumptions (or cost models) could lead to different cost results. If the taxi/cycler mass ratio is not a small value, then the V_∞ of encounters become important. In this case, the four-vehicle ballistic cyclers presented by Byrnes et al.[2] and McConaghy et al.[1] (which have relatively low V_∞) can provide important cost advantages. However, the cost associated with building and maintaining four cycler vehicles remains an important issue. When we consider that each vehicle might require substantial refurbishment on a regular basis, the cost of fewer vehicles becomes attractive. Suppose, for example, that 10% of the mass of each vehicle must be replaced or repaired every synodic period. In such a scenario the refurbishment cost is significantly lower for fewer vehicles.

We introduce a powered Earth–Mars cycler with a three-synodic-period repeat time. The new cycler requires three vehicles and provides some advantages to be considered with respect to previously studied cyclers.

Methodology

Before we patch together any semicyclers, suitable candidate trajectories must first be obtained. Let us consider those designed by Bishop et al.[9] and Aldrin et al.[13]

The Aldrin et al.[13] proposal includes two versions of semicyclers. In the first version (version I), the cycler vehicle leaves from an orbit about Mars, encounters the Earth twice, then returns to an orbit about Mars. (The trajectory sequence is hence MEEM, where E stands for Earth and M stands for Mars.) The transit times between the two planets range from about six months up to about nine months, and the entire sequence takes two synodic periods. A drawback of the version I semicycler is that the trajectory does not exist for every synodic opportunity as shown in Ref. 13. The second version (version II) is similar to the first one, except there are three Earth flybys separating the Mars departure and arrival (MEEEM). The time of flight between the first two Earth encounters is one year, whereas the time between the second and third Earth encounters is six months. A key feature of the version II semicycler is that it provides short time-of-flight (TOF) legs from Earth to Mars and from Mars to Earth. Version II semicyclers have Mars flyby V_∞ that range from about 2 to 7 km/s and Earth encounter V_∞ that range from about 3 to about 5 km/s. The entire sequence takes three synodic periods, thus requiring at least three vehicles to provide Earth–Mars and Mars–Earth transfers every synodic period.

In the Bishop et al.[9] proposal, the cycler vehicle leaves from an orbit around Mars (which requires a propulsive maneuver), encounters the Earth five times (separated by a year each), and then returns to Mars. Upon Mars arrival, the cycler vehicle performs a maneuver and is captured into orbit around Mars. The transit times between the two planets are about six months each, and the entire sequence completes in about five years. The Mars flyby V_∞ can range anywhere from about 3 to about 5 km/s, but the Earth encounter V_∞ can be as high as 9 km/s. A total of three cycler vehicles is needed to provide a transfer opportunity between Earth and Mars every synodic period (every 2.14 years). The main disadvantage of the Bishop semicycler is that the Earth flyby V_∞ are sometimes very high.

Out of these candidate semicyclers, we choose to patch together version II semicyclers into a cycler. Other semicyclers (such as that given by Bishop et al.[9]) can have merit in constructing cyclers, but these considerations will not be addressed in this paper.

Patching Semicyclers

To patch together series of version II semicyclers, we place the cycler vehicle in a heliocentric orbit instead of keeping it in a Mars parking orbit. (The amount of time that the cycler vehicle spends in deep space is the same as the waiting time that a semicycler spends at Mars—roughly two Mars years, or 1374 days.) The heliocentric orbit that we choose to use is a 3:2 resonance orbit with Mars (i.e., three spacecraft revolutions and two Mars revolutions about the sun). We note that this 3:2 resonance might not be optimal, but it is convenient for preliminary cycler construction. Our optimization program is capable of adjusting the TOF of this orbit (e.g., is capable of departing from our resonance initial guess) if needed. Although the semicyclers are launched from Mars, we assume that the cycler is launched from the Earth in the optimization of the trajectory.

Our patched cycler now has a flight sequence of EMMEE-EMMEE-EMMEE—. (The long dash means the sequence repeats as long as desired, and the short dashes separate the triple Earth encounters to enhance readability.) The time from the Earth launch (the first E) of the first EMMEE cycle to the next cycle is three Earth–Mars synodic periods (about 76 months, or 2340 days). The cycler thus has a repeat time of three Earth–Mars synodic periods (consistent with its semicycler source), and so a minimum of three vehicles is needed to provide transfer opportunities to and from each planet every synodic period.

To show that the cycler continues forever, we would have to computationally propagate the trajectory for an infinite amount of time, which, of course, is impossible. Instead we propagate the trajectory until the inertial positions of Earth, Mars, and the cycler vehicle repeat. The inertial geometry of Earth–Mars configuration approximately repeats every seven synodic periods or about 15 years. [Using 2.14 years as our definition of an Earth–Mars synodic period, our calculations show that the inertial geometry of Earth and Mars does not repeat exactly (in a sample calculation, the phase angle between the two planets changed by as much as 12 deg).] Our experience with the design of the cycler trajectory indicates that any change in the inertial orientation can be easily corrected by using low-thrust propulsion. Because our patched cycler has a repeat time of three synodic periods, at least seven repeat intervals (i.e., EMMEE sequences) are needed because the entire flight time must be an integer multiple of seven synodic periods. Therefore, we must design and compute at least 21 synodic periods (about 45 years) of patched cyclers to demonstrate the likelihood of perpetual repeatability. We recall that we will need a total of three such vehicles, each starting at a different mission time, in order to provide short-duration transfer opportunities (to Mars and to Earth) for every synodic period.

Reference 13 lists the trajectory itineraries for several version II semicyclers that provide us with good starting guesses. However, Ref. 13 does not have itineraries beyond the second repeat interval for any of the three vehicles; thus, we first calculate the remaining flyby dates based on trends observed from the listed data. We note that these calculations are by no means precise; the purpose they serve is to give us some reasonable starting guesses.

Circular-Coplanar Analysis

A circular-coplanar analysis can provide a rough estimate of what the cycler trajectory might look like. If the trajectory in the "circular-coplanar world" needs a deterministic maneuver, we can regard the required ΔV as a reasonable guess of the lower bound of the ΔV required in the "non-circular-coplanar world."

To calculate the cycler in the circular-coplanar model, we use a patched-conic method and assume that the Earth-to-Earth transfer is 1.5 years, the Mars-to-Mars transfer is 3.762 years (3:2 resonance), and the Earth-Mars and Mars-Earth transfers are both 0.572 years. The orbital radii of Earth and Mars are assumed to be 1 astronomical unit (AU) and 1.524 AU, respectively. The trajectory thus has a total repeat time of 6.406 years or three synodic periods. We then minimize the V_∞ at Earth and Mars to reduce taxi requirements. The resulting trajectory (Table 1) requires a V_∞ of 3.63 km/s with a flyby altitude of 5060 km at Earth and a 4.25 km/s V_∞ with a −645 km (subsurface) altitude flyby at Mars to continue ballistically. Because

Table 1 Circular-coplanar trajectory

Body	V_∞, km/s	ΔV, km/s	Altitude, km	TOF, days
E-1	3.63	0.00	5060	——
M-2	4.25	1.24	300	209
M-3	4.25	1.24	300	1374
E-4	3.63	0.00	5060	209
E-5	3.63	0.00	∞[a]	365
E-6[b]	3.63	0.00	5060	183

[a]Can be lowered by adjusting E-4 flyby altitude.
[b]Cycler trajectory repeats (E-6 = E-1).

subsurface flybys are not physically realizable, we replace the ballistic Mars encounter with a powered flyby that has a minimum altitude of 300 km and a ΔV of 1.24 km/s.

Low-Thrust Trajectory Optimization

The low-thrust trajectory optimizer we use is called the Gravity-Assist Low-Thrust Local Optimization Program[14–17] (GALLOP). GALLOP transforms the trajectory optimization problem into a nonlinear-programming (NLP) problem and maximizes the final spacecraft mass; it is driven by a sequential-quadratic-programming algorithm, SNOPT.[18]

The trajectory model in GALLOP divides each planet-planet leg of the trajectory into segments of equal duration. The thrusting on each segment is modeled by an impulse at the midpoint of the segment, with conic arcs between the impulses. Each leg is propagated halfway forward from the initial body and halfway backward from the final body. To have a feasible trajectory, one of the constraints that must be satisfied is that the forward- and backward-propagated half-legs must meet at a match-point time in the middle of the leg. The planetary positions and velocities are determined using an integrated (or analytic) ephemeris such as the Jet Propulsion Laboratory's DE405.

The optimization variables in GALLOP include the following: 1) the impulsive ΔV on each segment; 2) the Julian dates at the launch, flyby, and destination bodies; 3) the launch V_∞; 4) the incoming inertial velocity vectors at all of the postlaunch bodies; 5) the spacecraft mass at each body; 6) the flyby periapsis altitude at the gravity-assist bodies; and 7) the B-plane angle at the gravity-assist bodies. The optimization program can alter these variables to find feasible and optimal solutions of the given problem. A feasible solution means that the variables satisfy the constraints. These constraints include upper bounds on the impulsive ΔV on each of the segments, the launch-V_∞ magnitude, and the encounter dates at the bodies. Within the feasible set of solutions, the optimizer can find a solution that maximizes the final mass of the spacecraft.

Numerical Results: Concatenated Solution

In our first attempt, we were optimistic and tried to optimize the entire 21-synodic-period flyby sequence. The resulting numerical problem was enormous. Even with a rather coarse discretization size (30-day segments), our problem had 1968 optimization variables and 823 constraints (resulting in a matrix with 1,619,664 elements!). Because NLP run time increases rapidly with problem size, a problem of this size would take a long time to solve. Our first attempt took more than 13 h on our SunBlade 1000 Workstation, and the resulting trajectory was not even feasible (that is, at least one of the constraints was not satisfied). After several more attempts with no results to speak of, we decided that a new approach was needed.

The method we chose was to divide this gigantic problem (21 synodic periods, with flyby sequence of EMMEE-EMMEE-EMMEE—) into seven smaller problems (each one having a flyby sequence of EMMEEEM, lasting slightly over three synodic periods) and to optimize them separately. These smaller problems typically had about 300 optimization variables and about 120 constraints. The usual run time was around 10–20 min.

The seven smaller pieces ultimately must be connected back together to form a continuous cycler. To enforce this continuity, we overlapped the seven pieces with one another: the last two

Table 2 Vehicle 1 trajectory (concatenated solution)

Body	Date (mm/dd/yyyy)	V_∞, km/s	Altitude, km	TOF, days	ΔV, km/s
E-1	5/18/2007	9.893	——	——	——
M-2	1/10/2008	4.351	300[a]	237[b]	0.00
M-3	11/5/2011	4.664	300	1,395	0.12
E-4	6/6/2012	4.161	10,426	214[c]	0.00
E-5	5/28/2013	4.164	3,255	356	0.16
E-6	11/29/2013	4.281	18,579	185	0.01
M-7	7/16/2014	5.463	300	229[b]	1.84
M-8	5/5/2018	4.673	300	1,389	0.41
E-9	12/14/2018	6.147	12,471	223[c]	0.00
E-10	12/14/2019	6.130	300	365	0.00
E-11	6/12/2020	5.943	9,995	181	0.00
M-12	1/20/2021	4.445	300	222[b]	0.01
M-13	10/5/2024	5.534	488	1,354	1.30
E-14	6/3/2025	4.383	300	241[c]	0.00
E-15	6/3/2026	4.397	21,966	365	0.00
E-16	12/5/2026	4.526	2,894	185	0.00
M-17	8/27/2027	2.980	300	265[b]	0.23
M-18	1/18/2031	3.933	16,971	1,240	2.34
E-19	8/16/2031	4.302	300	210[c]	0.00
E-20	8/15/2032	4.287	300	365	0.00
E-21	2/12/2033	4.397	300	181	0.00
M-22	12/10/2033	4.754	300	301[b]	0.26
M-23	9/6/2037	6.426	300	1,366	0.93
E-24	3/23/2038	4.453	22,448	198[c]	0.54
E-25	9/26/2038	4.427	1,914	187	0.00
E-26	9/26/2039	4.408	1,012	365	0.00
M-27	4/15/2040	4.714	17,185	202[b]	0.40
M-28	8/1/2043	2.747	300	1203	1.66
E-29	7/28/2044	4.253	10,383	362[c]	0.00
E-30	7/29/2045	4.243	30,957	366	0.00
E-31	1/26/2046	4.376	35,809	181	0.00
M-32	11/12/2046	4.761	300	290[b]	0.31
M-33	8/17/2050	5.176	300	1,374	1.18
E-34	3/26/2051	6.948	37,279	221[c]	0.78
E-35	3/25/2052	6.941	300	365	0.00
E-36	9/28/2052	6.909	413	187	0.00
M-37	7/29/2053	3.635	——	304[b]	0.00
——	——	5.180 (average Earth)	——	——	12.31 (total)
——	——	4.550 (average Mars)	——	——	0.58 (per synodic period)

[a]An altitude constraint of 300 km is assumed at both Earth and Mars. [b]Outbound transfer time from Earth to Mars. [c]Inbound transfer time from Mars to Earth.

Table 3 Vehicle 2 trajectory (concatenated solution)

Body	Date (mm/dd/yyyy)	V_∞, km/s	Altitude, km	TOF, days	ΔV, km/s
E-1	2/8/2003	5.596	——	——	——
M-2	9/22/2003	4.538	300[a]	226[b]	1.19
M-3	6/23/2007	4.274	300	1,370	0.20
E-4	2/18/2008	5.255	17,706	240[c]	0.39
E-5	2/17/2009	2.273	327	365	1.00
E-6	8/22/2009	5.153	300	186	0.51
M-7	4/8/2010	6.579	542,759	229[b]	1.79
M-8	8/30/2013	2.806	778	1,240	2.02
E-9	9/7/2014	5.800	19,429	373[c]	0.00
E-10	9/7/2015	5.789	1,138	365	0.00
E-11	3/5/2016	5.882	31,308	180	0.00
M-12	10/13/2016	5.269	300	222[b]	1.79
M-13	7/12/2020	5.179	5,594	1,368	0.17
E-14	2/23/2021	5.758	310	226[c]	0.00
E-15	2/23/2022	5.775	9,982	365	0.00
E-16	8/28/2022	5.655	16,780	186	0.00
M-17	5/20/2023	3.446	300	265[b]	0.65
M-18	7/4/2026	2.879	300	1,141	1.87
E-19	5/27/2027	3.236	300	327[c]	0.06
E-20	5/10/2028	3.152	10,289	349	0.23
E-21	11/12/2028	3.217	19,647	186	0.00
M-22	9/2/2029	3.063	11,643	294[b]	0.23
M-23	5/4/2033	3.858	300	1,340	1.36
E-24	12/9/2033	6.197	16,470	219[c]	0.00
E-25	12/19/2034	3.329	9,021	375	1.35
E-26	6/19/2035	3.224	5,369	182	0.00
M-27	1/7/2036	2.685	8,729	202[b]	0.01
M-28	8/20/2039	3.891	300	1,321	1.96
E-29	4/26/2040	3.956	3,487	250[c]	0.00
E-30	4/15/2041	3.278	9,115	354	0.43
E-31	10/19/2041	3.301	1,541	187	0.00
M-32	8/5/2042	2.910	6,157	290[b]	0.04
M-33	4/16/2046	4.293	300	1,350	1.62
E-34	12/22/2046	9.960	300	250[c]	0.45
E-35	12/22/2047	9.975	300	365	0.00
E-36	6/21/2048	9.654	5,441	182	0.00
M-37	4/21/2049	8.029	——	304[b]	1.20
——	——	5.246 (average Earth)	——	——	22.03 km/s (total)
——	——	4.247 (average Mars)	——	——	1.05 km/s (per synodic period)

[a]An altitude constraint of 300 km is assumed at both Earth and Mars. [b]Outbound transfer time from Earth to Mars. [c]Inbound transfer time from Mars to Earth.

encounters (i.e., EM) of each EMMEEEM piece are the first two bodies of the next EMMEEEM piece. Furthermore, we constrained the incoming V_∞ vectors to be the same at the overlapping Mars (i.e., the M of each EM) of two consecutive EMMEEEM pieces. We were thus able to construct all three cycler trajectories.

Summaries of the resulting trajectories of the three required vehicles are listed in Tables 2–4. The numbering of the vehicles is completely arbitrary and has no significance besides clarifying references to a particular vehicle.

A portion of the trajectory plot is shown in Fig. 1. We note that it is remarkably similar to the trajectory plot of a version II semicycler.[13] (A complete trajectory plot would be too cluttered because of so many flybys.) Also, because of a software limitation, the trajectory appears to be connected by straight lines, but in actuality it is connected by conic arcs. In Fig. 2, we use a radial distance plot, which plots the distance of the spacecraft from the sun vs time since launch. Figure 2 shows a portion of a typical radial distance plot for vehicle 2. (This portion corresponds to E-1–E-11 for vehicle 2.) The vertical lines mark sun–Earth–Mars oppositions (and thus the time interval separating the vertical lines correspond to an Earth–Mars synodic period). We note in Fig. 2 that the first Mars–Mars transfer (between the 280th and 1600th day) is a 3:2 resonance, whereas the second Mars–Mars transfer is not; this is a good example of the optimizer finding a nonresonance transfer that is better than a resonance orbit. The fact that GALLOP can take an initial guess that contains resonance transfers and alter it is worth noting. Gravity-assist trajectories that include resonance transfers can sometimes lead to numerical problems, as there are essentially an infinite number of solutions, each differing in the inclination. In fact, glancing

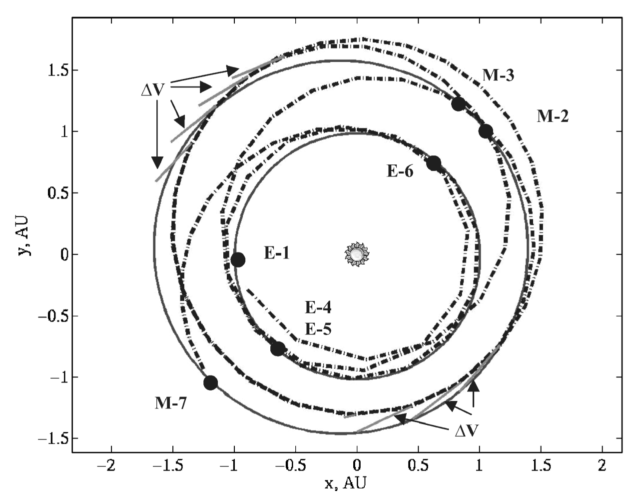

Fig. 1 Representative partial trajectory plot (E-1–M-7 for vehicle 3).

at Tables 2–4 indicates that most of the Mars–Mars potions of the cycler are nonresonant (whenever the Mars–Mars time of flight is not 1374 days). In Fig. 3 we show a portion of the ΔV profile for vehicle 3 (E-1–E-6). The power availability constraint is the maximum ΔV that the spacecraft can use corresponding to the available solar power to the engine as the spacecraft changes it distance from the sun. The majority of the thrusting occurs on the Mars-to-Mars legs and close to the Mars encounters. The primary purpose of the

Table 4 Vehicle 3 trajectory (concatenated solution)

Body	Date (mm/dd/yyyy)	V_∞, km/s	Altitude, km	TOF, days	ΔV, km/s
E-1	3/23/2005	8.422	——	——	——
M-2	11/3/2005	4.391	300[a]	225[b]	0.00
M-3	8/29/2009	3.243	5,147	1,395	2.86
E-4	5/10/2010	3.250	17,144	254[c]	0.00
E-5	5/11/2011	3.232	3,820	366	0.00
E-6	11/12/2011	3.296	17,278	185	0.00
M-7	7/8/2012	4.335	6,343	239[b]	0.23
M-8	1/14/2016	3.226	300	1,285	0.86
E-9	11/24/2016	7.994	9,053	315[c]	0.00
E-10	11/25/2017	8.008	303	366	0.00
E-11	5/24/2018	7.806	19,702	180	0.00
M-12	2/12/2019	5.895	300	264[b]	0.01
M-13	10/14/2022	7.297	300	1,340	2.18
E-14	5/9/2023	5.364	1,086	207[c]	0.00
E-15	5/8/2024	5.363	191,947	365	0.00
E-16	11/10/2024	5.468	853	186	0.00
M-17	8/12/2025	2.484	84,256	275[b]	0.13
M-18	2/10/2029	3.926	300	1,278	3.48
E-19	11/14/2029	8.629	21,976	277[c]	1.00
E-20	12/2/2030	5.901	300	383	1.44
E-21	5/31/2031	5.739	300	180	0.00
M-22	4/5/2032	7.321	300	310[b]	0.14
M-23	8/24/2035	5.633	1,932	1,236	3.54
E-24	3/28/2036	6.209	2,313	217	0.00
E-25	3/28/2037	6.205	131,919	365	0.00
E-26	10/1/2037	6.189	300	187	0.00
M-27	6/16/2038	2.560	300	258[b]	0.01
M-28	11/6/2041	4.573	2,062	1,239	1.23
E-29	8/9/2042	6.665	18,927	276[c]	0.00
E-30	8/7/2043	6.977	300	363	0.20
E-31	2/4/2044	7.178	300	181	0.00
M-32	11/14/2044	5.682	1,192	284[b]	0.01
M-33	6/12/2048	4.423	300	1,306	0.30
E-34	2/3/2049	8.881	307	236[c]	0.00
E-35	2/4/2050	8.880	4,735	366	0.00
E-36	8/7/2050	8.631	5,636	184	0.00
M-37	6/11/2051	6.410	——	308[b]	0.00
——	——	6.559 (average Earth)	——	——	17.62 (total)
——	——	4.760 (average Mars)	——	——	0.83 (per synodic period)

[a]An altitude constraint of 300 km is assumed at both Earth and Mars. [b]Outbound transfer time from Earth to Mars. [c]Inbound transfer time from Mars to Earth.

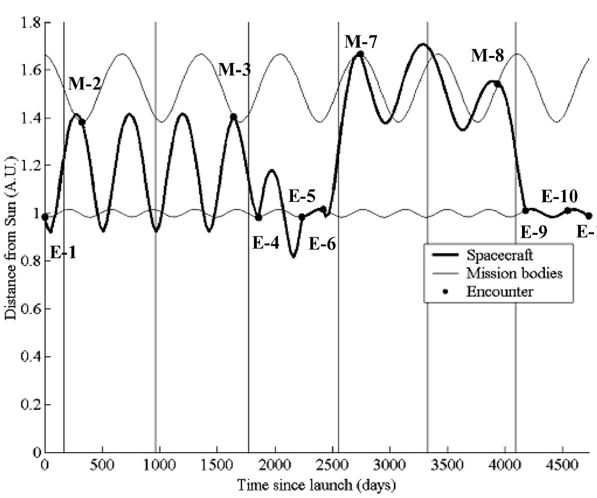

Fig. 2 Partial radial distance plot (E-1–E-11 for vehicle 2).

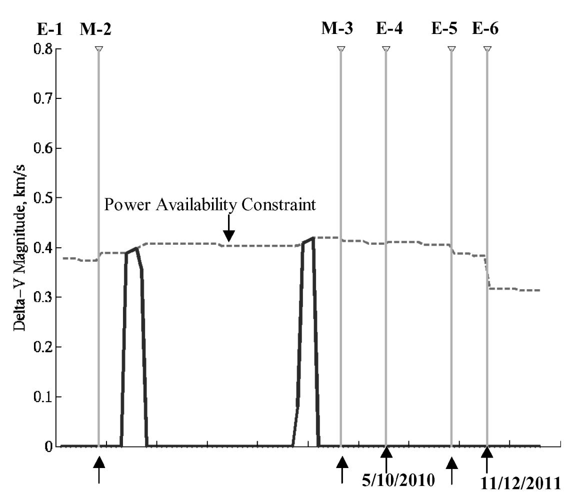

Fig. 3 Partial ΔV profile (E-1–E-6 for vehicle 3).

Mars-to-Mars thrusting arcs is to augment the insignificant amount of bending and inclination-change that a Mars gravity assist can provide. Sometimes thrusting arcs occur during the triple-Earth encounters, and these maneuvers are used to correct the spacecraft motion when conditions for resonance and half-revolution transfers are not met.

The results in Tables 2–4 each cover the entire 21-synodic-period time frame. After any integer multiple of seven Earth–Mars synodic periods, the inertial alignments of the two planets (essentially) repeat, and so we can predict that the next set of 21 synodic periods of the cycler will be similar to the results in Tables 2–4. Thus we have also demonstrated that the cycler has a chance to repeat perpetually, provided that propellant is available to make up for the differences in the V_∞ between E-1 and E-36 for all three vehicles. (These differences in V_∞ are partly caused by the nonexact inertial realignment of Earth and Mars and partly caused by boundary effects.) Even if the cycler does not repeat exactly after 21 synodic periods, our experience has shown that extending the cycler trajectory is just a matter of constructing the next seven (or more) repeat intervals using the same methodology. We can thus construct the cycler for an arbitrarily long period of time.

As shown at the bottom of Tables 2–4, the cumulative ΔV for the three vehicles range from about 0.58 to 1.05 km/s per synodic period. The ΔV figures translate to an average total usage of about 2.43 km/s per synodic period. It is interesting (and reassuring) to note that this value agrees remarkably well with the estimation using a circular-coplanar analysis, described earlier in this paper.

Numerical Results: Optimal Solution

With our three concatenated solutions in hand, we once again turned to the problem of the optimization of the complete (21-synodic-period) trajectory. We expected that the concatenated solutions would provide initial guesses that are good enough for the optimization of the whole trajectory. To further speed up computation, we also reduced the number of segments between each Mars-to-Mars leg. (The numbers of segments between all of the other legs were kept the same as in the concatenated solutions.)

Only the most crucial information (flyby dates, altitudes, and B-plane angles) was transferred to the input file for the optimal solution. (Ideally, we would include the thrust profiles also, but because of a software limitation the inclusion of the thrust profiles would have taken an impractically long time.) Using a concatenated solution as an initial guess, we were able to obtain a complete (that is, the entire 21 synodic period) optimal trajectory for vehicle 1, after numerous tries. Vehicles 2 and 3, however, proved to be much harder, and we were unable to achieve optimal solutions for either (after about 100 person-hours were devoted to the attempt).

Table 5 shows the optimal-trajectory itinerary for vehicle 1 (in which the final vehicle mass is maximized). Comparing the data in Table 5 to those in Table 2 (the concatenated solution), we note several differences. First, all of the dates have moved, some by as much as two months. Second, we see that the average V_∞ at both Earth and Mars have dropped by nearly half a kilometer per second. Lastly, and perhaps the most perplexing differences, is that the concatenated solution has a lower total ΔV. One would expect to see that the "piecewise optimal" solutions have somewhat higher ΔV than the whole solution, whose "patch point" conditions are also optimized. One reason for this discrepancy is that the larger segments on the Mars–Mars legs might have constrained the solution to require higher ΔV. We recall that to expedite the optimization process we decreased the number of segments between the Mars encounters (thus lengthening each segment), and we reduced the places where thrusts are allowed. These restrictions on the thrust locations might have forced the optimizer to choose locations that are suboptimal, resulting in the slightly higher total ΔV.

However, there is considerable agreement between the circular-coplanar, the concatenated, and the optimal results for vehicle 1. The circular-coplanar analysis shows a ΔV cost of 2.48 km/s per synodic period (for all three vehicles combined), whereas Tables 1–3 give an average ΔV cost of 2.43 km/s per synodic period. The optimal result, seen in Table 5 (for vehicle 1 alone), gives a ΔV expenditure of 0.64 km/s per synodic period, which is in close agreement with the value of 0.58 km/s per synodic period from Table 2. Assuming that the optimal results of the other two vehicles are also similar to their respective concatenated solutions, the total ΔV cost for

Table 5 Vehicle 1 trajectory (optimal solution)

Body	Date (mm/dd/yyyy)	V_∞, km/s	Altitude, km	TOF, days	ΔV, km/s
E-1	5/14/2007	9.893	——	——	——
M-2	1/16/2008	3.985	1,925	247[b]	0.00
M-3	7/23/2011	2.921	300[a]	1,283	0.98
E-4	5/30/2012	3.862	300	312[c]	0.00
E-5	5/30/2013	3.861	938,502	365	0.00
E-6	12/1/2013	3.972	28,679	185	0.00
M-7	8/8/2014	4.286	809	251[b]	0.00
M-8	3/3/2018	3.189	300	1,303	0.81
E-9	12/8/2018	5.068	28,517	280[c]	0.00
E-10	12/8/2019	5.073	782	365	0.00
E-11	6/6/2020	4.923	300	181	0.00
M-12	12/10/2020	3.988	300	187[b]	0.00
M-13	8/15/2024	3.833	300	1,344	1.37
E-14	4/20/2025	3.630	300	248[c]	0.08
E-15	4/11/2026	3.969	300	356	0.32
E-16	10/14/2026	3.989	764	186	0.00
M-17	6/23/2027	4.283	300	252[b]	0.67
M-18	2/13/2031	4.353	300	1,331	3.66
E-19	9/15/2031	5.748	7,728	214[c]	0.60
E-20	8/29/2032	4.483	1,900	350	0.93
E-21	2/25/2033	4.574	300	179	0.00
M-22	1/1/2034	5.056	14,998	310[b]	0.09
M-23	9/27/2037	7.236	1,472	1,364	1.67
E-24	4/4/2038	4.439	300	190[c]	0.38
E-25	10/8/2038	4.441	525,292	187	0.00
E-26	10/8/2039	4.442	10,149	365	0.00
M-27	5/19/2040	3.349	14,322	224[b]	0.00
M-28	8/3/2043	2.444	300	1,171	1.64
E-29	7/11/2044	3.489	4000	343[c]	0.17
E-30	7/12/2045	3.492	348,584	365	0.00
E-31	1/10/2046	3.610	46,408	182	0.00
M-32	10/25/2046	3.949	675	288[b]	0.00
M-33	7/17/2050	4.326	300	1,361	0.07
E-34	3/20/2051	6.563	38,684	246[c]	0.00
E-35	3/20/2052	6.544	1,094	365	0.00
E-36	9/22/2052	6.497	433	186	0.00
M-37	7/1/2053	3.218	——	282[b]	0.00
——	——	4.843 (average Earth)	——	——	13.44 (total)
——	——	4.028 (average Mars)	——	——	0.64 (per synodic period)

[a]An altitude constraint of 300 km is assumed at both Earth and Mars. [b]Outbound transfer time from Earth to Mars. [c]Inbound transfer time from Mars to Earth.

the three optimal trajectories will likely be near the values of the circular-coplanar analysis.

Discussion

We must admit that there are a number of shortcomings to the powered cycler. The propellant expenditure is higher than many of the other cyclers in the literature. We also note that some of the transfer times (both Earth-to-Mars and Mars-to-Earth) of our cycler are not as short as we would have liked. For Earth–Mars transfers, the TOF range from 6.5 to 12 months, with an average of 8.4 months. [In comparison, the Aldrin cycler has Earth–Mars TOF, which range from 4.7 to 5.6 months, with an average of 5.2 months. The four-vehicle cycler (S1L1 of McConaghy et al.[1]) TOF range from 3.8 to 7.2 months and average 5.3 months.]

For Mars–Earth transfers, the TOF for our powered cycler are between 6.7 and 10 months, with an average of 8 months. (The Aldrin cycler's Mars–Earth TOF, on the other hand, range from 4.7 to 5.6 months, with an average of 5.1 months. The four-vehicle cycler has values that range from 3.8 to 7.5 months, with an average of 5.4 months.) For human transportation (a reasonably optimistic guess of the first human application is perhaps around the year 2031), the desired TOF is probably around 6 months. However, in terms of V_∞ our powered cycler compares favorably to the Aldrin cycler, especially at Mars. The Aldrin cycler has an average Earth V_∞ (for the Earth–Mars transfer) of 5.86 km/s, whereas our powered cycler's average Earth V_∞ (the average of all three vehicles) is 5.66 km/s. At Mars, the Aldrin cycler's average V_∞ is 9.10 km/s (for the Mars–Earth transfers), and the average value for our powered cycler is 4.52 km/s. For the four-vehicle cycler, the V_∞ at Earth range from 3.99 to 7.02 km/s, for an average of 5.54 km/s; the V_∞ at Mars range from a low of 2.76 km/s to a high of 7.72 km/s and an average of 5.15 km/s.

The long Mars-to-Mars transfers (used to replace parking orbits at Mars) of our cycler can present some challenges if the cycler were to be used in a human transportation system. In such a system, the Mars-to-Mars transfers would most likely be unoccupied and would complicate the maintenance and upkeep of the cycler vehicle. (We note that such unoccupied flight in deep space is an inherent shortcoming of all cyclers.) However, we do not rule out the possibility of the cycler vehicle being used during these Mars–Mars transfers, perhaps by scientists and researchers, or maybe even adventurous individuals.

Conclusions

We introduce a three-vehicle-powered cycler, which provides some important design advantages. In our proposed cycler, each vehicle provides both an outbound and inbound transfer leg, in distinction to the known two-vehicle and four-vehicle cyclers, which only provide one transfer leg (either outbound or inbound, but not both). The three-vehicle cycler has lower average V_∞ (at Earth and at Mars) than the Aldrin cyclers, but higher than that of the four-vehicle cycler (at Earth).

Designing cyclers is a formidable task, particularly in the powered case described in this paper. Even with the latest software techniques for low-thrust gravity-assist trajectory optimization, this numerically challenging problem was still not solvable without compromise.

The new powered cycler that we have presented is most likely not optimal (though we think it is close to optimal), but the trajectory is certainly feasible and flyable, with reasonable expenditure of propellant. We have shown that there is good reason to believe that this cycler can be propagated into the distant future.

Finally, we hope that this powered cycler, which requires only three vehicles to provide complete coverage of all synodic transfer opportunities, will prove a useful benchmark in future Mars transportation architecture studies.

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